AD-A117 910 NAVAL SURFACE WEAPONS CENTER DAHLGREN VA F/G 3/1
MEAN EARTH ELLIPSOID DETERMINED FROM SEASAT ALTIMETER OBSERVATI--ETC(U)
SEP 81 G B WEST
UNCLASSIFIED NSWC-TR-252 NL

1 of 1

END
DATE
FILMED
8 82
DTIC

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 81-252	2. GOVT ACCESSION NO. AD-A117910	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MEAN-EARTH ELLIPSOID DETERMINED FROM SEASAT ALTIMETER OBSERVATIONS		5. TYPE OF REPORT & PERIOD COVERED Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Gladys B. West		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center (K12) Dahlgren, VA 22448		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 63701B
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Mapping Agency Washington, DC 20360		12. REPORT DATE September 1981 13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Mean-earth ellipsoid
Altimetric geoid
Geoid heights
Ellipsoidal parameters
Along-track geoid heights

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The radius and the origin of a mean-earth ellipsoid have been determined from SEASAT satellite radar altimeter observations of the ocean surface and potential coefficients. Assuming that the altimetric geoid is the best available, adjustments are made to the geoid derived from potential coefficients so that the geoid height differences are minimized. A least-squares fitting procedure based on mean geoid heights in 5° x 5° areas was used in the determination of the ellipsoidal parameters. Solutions from two different sets of altimetric geoid heights and geoid heights derived from potential coefficients

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. ABSTRACT (Continued)

gave a mean-earth ellipsoid with radius, a = 6378134.9 m and origin shifts less than 1 m in the x and y components and approximately -2.5 m in the z component.

FOREWORD

The radar altimeter aboard the SEASAT satellite, launched 26 June 1978, provided measurements over the oceans from which along-track geoid heights were derived. These values, along with geoid heights from potential coefficients, are used in the determination of the radius and origin shifts of a mean-earth ellipsoid. R. J. Anderle of the Naval Surface Weapons Center (NSWC) has done extensive work on the accuracy of a mean-earth ellipsoid based on Doppler, laser, and altimeter observations. The results presented in this report were used in his work. The project was conducted under Geodetic and Geophysical Support Project DMA PE 63701B/3204/240.

This report was reviewed by Carlton W. Duke, Jr., Head, Space and Surface Systems Division and Ralph L. Kulp, Jr., Head, Space and Ocean Geodesy Branch.

Released by:

C.A. Fisher

C. A. FISHER, Head
Strategic Systems Department

CONTENTS

INTRODUCTION

Geoid heights derived from SEASAT satellite altimeter observations of the ocean surface and from potential coefficients were used in a least-squares solution for determination of the radius and origin shifts of a mean-earth ellipsoid. This report gives a description of the geoid height data used in the solutions for ellipsoid parameters, results of the solutions, and summary.

SEASAT ALTIMETER OBSERVATIONS

The altimeter observations were collected by the SEASAT satellite that was launched 26 June 1978 and recorded data continuously over the oceans. The mission ended 9 October 1978 after approximately 1000 revs of altimeter observations were obtained. These observations were prefiltered using a linear fit and a fixed sigma as criteria for discarding possible extraneous points. The prefiltered observations given at 10 points per second (pps) were averaged to 2 pps and reduced to best-estimated along-track altimeter geoid heights, N_A. Further editing included examining data at both ends of time continuous segments and eliminating geoid heights with corresponding vertical deflections outside the range ±60 arc sec. The reduction of the altimeter measurements to geoid heights entailed making atmospheric and environmental corrections, combining with the precise orbital ephemeris (Colquitt et al., 1980) and applying a Kalman smoother (West et al., 1977). The ephemeris was computed using SEASAT Doppler observations. Two sets of altimeter geoid heights (Table 1) were selected for the ellipsoid parameter solutions from the available revs of data. The altimeter geoid heights are based on a reference ellipsoid with a radius of 6378145 m and reciprocal flattening of 298.26. A plot of the subtracks for both altimeter data sets are given in Figures 1 and 2.

The along-track geoid heights were averaged to give a mean geoid height N_A for each 5° x 5° area. The earth's surface was divided into 2160 5° x 5° areas bounded by latitude lines ±75° and longitude lines 0° and 360°. N_A is assumed to be located at the center of the 5° x 5° area.

GRAVITY DATA FROM SATELLITE OBSERVATIONS

Station coordinates, station heights above mean sea level, and potential coefficients derived from satellite observations were used in the determination of the separation of the mean sea level from the reference ellipsoid, the geoid height (Groeger, 1968). This separation is referred to here as the geoid height derived from potential coefficients, N_s. A subset of NWL9D station coordinates and heights (Table 2) and the DoD WGS72 (Seppelin, 1974) potential coefficients were used in the generation of N_s at 1° x 1° grid points. The geoid heights are based on a reference ellipsoid with a radius

of 6378135 m and flattening reciprocal of 298.26. Groeger also provides an adapted reference ellipsoid with a radius of 6378138.58 m and flattening reciprocal of 298.28 that best represents the equipotential geodial surface.

Table 1. Revs of SEASAT Altimeter Data

Set 1		Set 2	
Day	Revs	Day	Revs
207	416-428	211	472-473
			475-477
			479-486
208	429-443		
209	444-457	212	487-498
			500
215	530-541	218	573-576
			579-584
216	544-557		
217	558-570	219	587-597
	572		600
220	601-611	227	707-715
	613-614	228	716-727
			729
221	615-627		
222	631-640	232	773-784
	643		786
224	661-667	233	787-801
	669-670		
		234	802-812
225	673-686		815
		235	816-819
			823-827
			829
		236	830-832

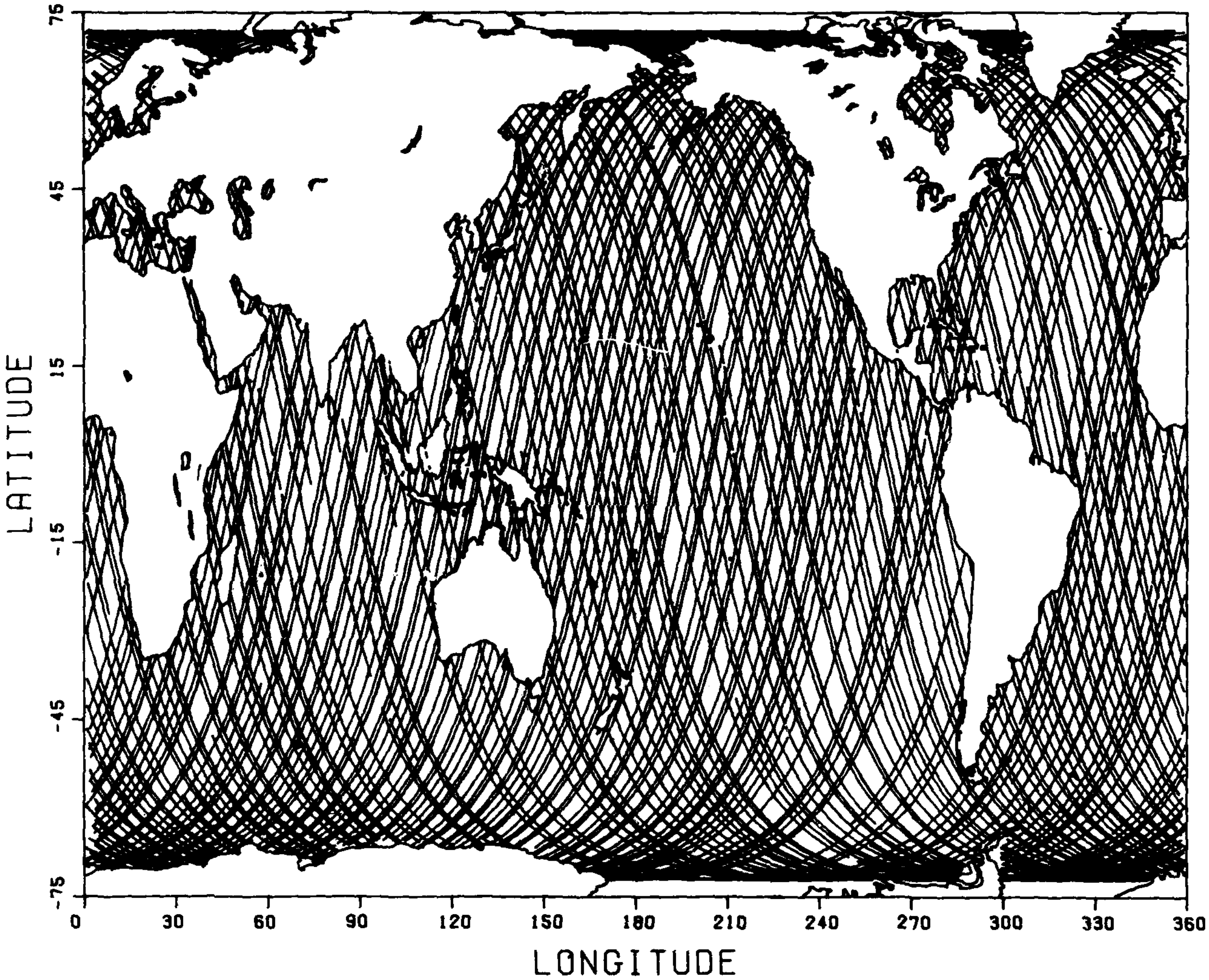


Figure 1. 142 Revs of SEASAT Altimeter Data Selected from Days 207 to 225

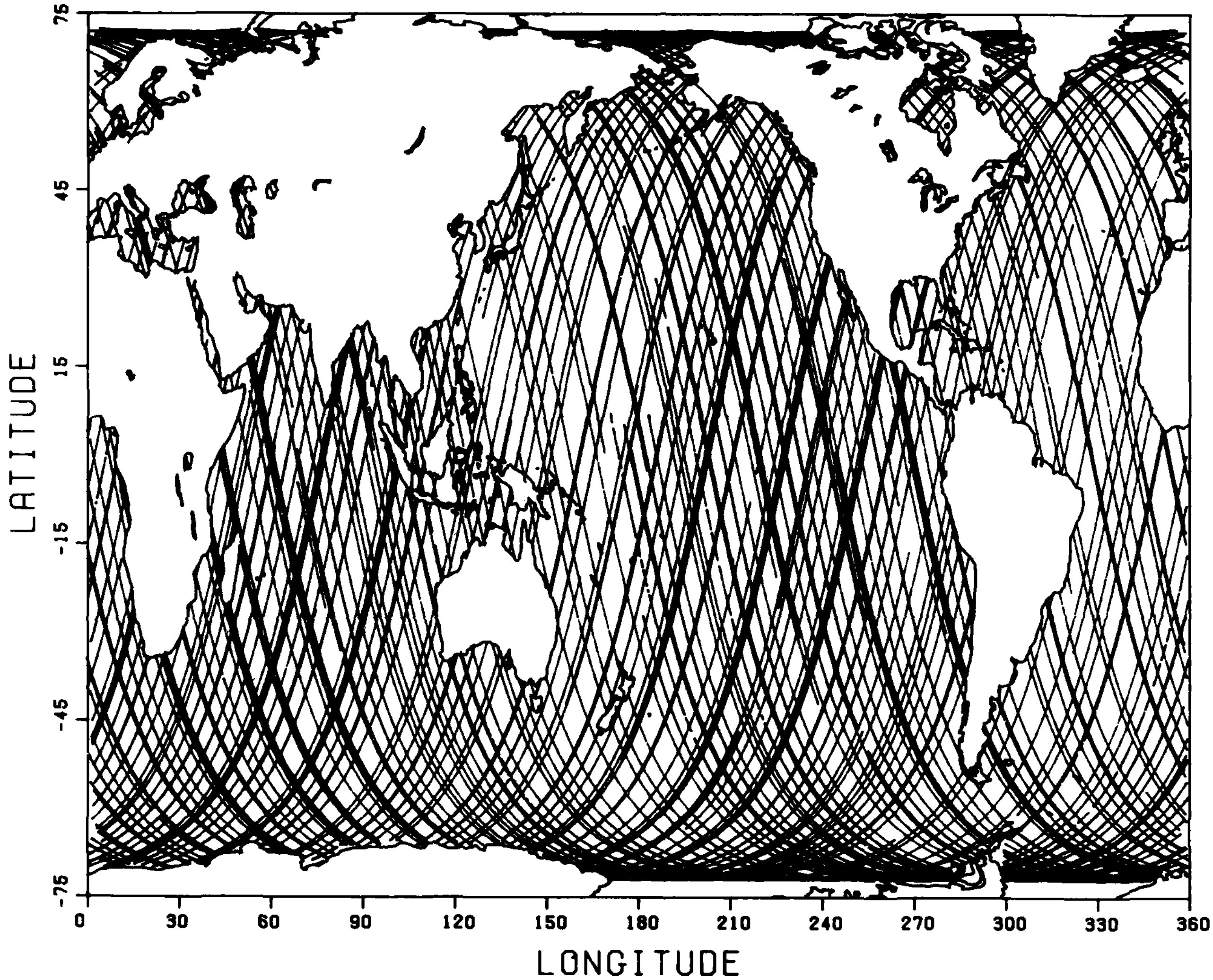


Figure 2. 123 Revs of SEASAT Altimeter Data Selected from Days 211 to 236

Table 2. Station Positions and Heights (NWL9D)

Station Number	Location	Latitude[1]	Longitude	hms[2](m)	he[1](m)
8	Brazil	-23°13'	314° 7'	612.68	605.27
19	McMurdo	-77°50'	166°40'	38.2	-17.84
20	Seychelles	-4°40'	55°28'	592.5	546.14
21	Uccles	50°47'	4°21'	115.8	148.16
22	Philippines	14°59'	122° 4'	11.5	49.17
23	Guam	13°26'	144°38'	38.18	84.58
24	Samoa	-14°19'	189°17'	9.2	39.07
27	Japan	39° 8'	141° 7'	82.6	114.12
105	Pretoria	-25°56'	28°20'	1580.56	1600.87
107	Virginia	38°59'	282°41'	118.6	76.86
112	Australia	-34°40'	138°39'	34.44	31.35
113	New Mexico	32°16'	253°14'	1205.84	1172.78
114	Alaska	61°17'	210°10'	67.6	67.27
116	England	51°11'	358°37'	78.93	115.94
118	Greenland	76°32'	291°14'	54.79	59.14
128	Ottawa	45°23'	284° 4'	86.08	40.57
192	Texas	30°23'	262°16'	178.5	208.12
311	Maine	44°24'	291°59'	25.88	-11.68
320	Minnesota	44°43'	266°55'	299.5	259.75
330	California	34° 6'	240°56'	461.56	402.18
340	Hawaii	21°31'	202° 0'	401.19	407.37
641	Florence	43°48'	11°13'	100.3	137.38
9943	Orroral	-35°37'	148°57'	929.53	942.56
30078	Massachusetts	42°37'	288°30'	140.55	104.15
30120	La Paz	-16°31'	291°49'	4041.93	4085.47
30121	Quito	- 0° 5'	281°34'	2682.59	2704.0
30122	Paraguay	-25°18'	302°23'	177.16	1893.86
20284	Sicily	37°24'	14°56'	20.83	534.73
30124	Tehran	35°44'	51°23'	1421.22	1415.76
30126	Zaire	-4°22'	15°15'	453.88	448.63
30130	Cyprus	35° 0'	33°43'	101.9	118.42
30188	Hawaii	21°18'	202° 0'	4.01	9.63
30203	Kenya	-1°19'	36°48'	1677.31	1656.69
30414	Calgary	50°52'	245°42'	1267.8	1238.22
30793	Townsville	-19°15'	146°45'	6.41	581.96
30800	Thailand	13°47'	100°35'	15.34	-22.32
30967	Bermuda	32°19'	295° 9'	24.3	-13.88

[1] Geodetic coordinates with respect to an ellipsoid having a radius of 6378145 m and a reciprocal flattening of 298.25

[2] Height above mean sea level

Mean geoid heights, N_s, were obtained for each 5° x 5° area by averaging the N_s values at the 1° x 1° grid points contained within the area. N_s is referenced to location ϕ, λ at the center of each 5° x 5° area. The total surface area is the same as that described earlier for the altimetric geoid heights. Geoid heights were similarly derived from the GEM10B (Lerch, 1978) potential coefficients using the same station coordinates and heights.

SOLUTIONS FOR RADIUS AND ORIGIN OF A MEAN-EARTH ELLIPSOID

Ellipsoid parameters, radius, and the origin of a mean ellipsoid best fitting the mean geoid height differences (ΔN) obtained from altimetric observation and potential coefficients are solved for. The mean geoid heights were derived using different reference ellipsoids. If it is assumed that the 'tim- etric geoid from the ocean areas is the best available, then in order tc ..d the equipotential geoidal surface (as defined by potential coefficients) 'at best fits the difference between the altimetric and potential coefficier geoids, adjustments are made to the geoid derived from potential coeffic 's such that the mean geoid height difference is zero. Figure 3 shows the sets of geoid heights and the associated reference ellipsoids. N_A is th mean altimetric geoid height located at the center of the i^{th} 5° x 5° square and based on reference ellipsoid E_A. N_S, the mean geoid height derived from potential coefficients, is also located at the center of the i^{th} 5° x 5° square and based on reference ellipsoid E_S. G_A and G_S are the altimetric and potential coefficients geoids, respectively.

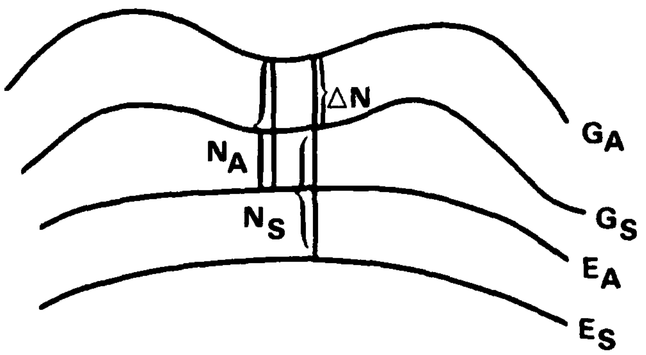


Figure 3. Altimeter and Potential Coefficients Geoid Heights

A change in geoid heights from different reference ellipsoids and a shift in the ellipsoid origin may be expressed as given in Equation 1 (Heiskanen and Moritz, 1967)

$$N_A = N_S + \frac{\partial N}{\partial a}\Delta a + \frac{\partial N}{\partial x}\Delta x + \frac{\partial N}{\partial y}\Delta y + \frac{\partial N}{\partial z}\Delta z \qquad (1)$$

where Δa, Δx, Δy, and Δz are increments in the ellipsoid radius and origin solved for in a least-squares fit (see Appendix A) to the geoid height differences.

The adjustment $[\Delta a + (a_A - a_S)]$ derived from the solution is applied to the radius, "a_1" of the reference ellipsoid that best fits the geoid from potential coefficients. Δa is the discrepancy between the altimetric geoid and the geoid derived from potential coefficients, and $(a_A - a_S)$ is the difference between the radii of the reference ellipsoids for the altimetric and potential coefficients data, respectively.

The mean ellipsoid with radius "a" that best fits the equipotential geoidal surface derived from potential coefficients is given below by Equation 2.

$$a = a_1 + [\Delta a + (a_A - a_S)] \qquad (2)$$

The results of the solutions are shown in Table 3. The first solution was derived from geoid heights based on WGS72 potential coefficients and altimetric data. The altimetric geoid heights were taken from 142 revs of satellite data selected from the time period from day 207 to 225. Solution 1 is weighted by the number of data points in each 5° x 5° square. Solution 2 is a repeat of Solution 1 with the exception that geoid heights derived from WGS72 potential coefficients were replaced by those obtained from GEM10B potential coefficients. The approximate 1 m discrepancy in the radius, a, of the mean-earth ellipsoid as shown between Solutions 1 and 2 is due to the difference in potential coefficients.

Table 3. Ellipsoid Parameter Solutions in Meters

Solution	Reference Geoid	Number of Points	Δx	Δy	Δz	a
1	WGS72	992,602	-0.09	0.12	-2.47	6378134.30
2	GEM10B	992,602	0.01	0.19	-2.55	6378135.43
3	WGS72	884,453	-0.33	-0.17	-2.67	6378134.17
4	GEM10B	884,453	-0.22	-0.07	-2.65	6378135.29

Solutions 3 and 4 were computed from the second set of altimetric data (Table 1) and also weighted by the number of points in each 5° x 5° area.

SUMMARY

The earth's radii derived from the above solutions were found to be consistent between the two sets of altimetric data both of which contained mean geoid heights in more than 1750 5° squares. The WGS72 and GEM10B potential coefficients gave a difference of 1.1 m in the radii. An average of the results from the two sets of potential coefficients shows the reference ellipsoid that best fits the equipotential geoidal surface derived from potential coefficients has a radius of 6378134.9 m. The results presented in this report are corrected results to those given under the heading "SEASAT-1 Altimetric Value for Earth's Semimajor Axis (West)" by Anderle (1980).

The earth's radius has been derived by others (Anderle, 1980) from several data sets with results that range from 6378139 ±1m derived from laser station positions to 6378134.5 m derived from GEOS-3 satellite altimetry data. The radius of the mean earth reference ellipsoid obtained here falls within the above range. The results of this study contributed to the value of 6378136 m recommended as a standard value by Anderle (1980) to the International Union of Geodesy and Geophysics in December 1979. At this meeting a new Geodetic Reference System 1980 with the radius of the earth, a = 6378137 m, was adopted.

The origin shifts shown in Table 3 are less than 1 m for the x and y components. An average of the shifts in the z components from both WGS72 and GEM10B potential coefficients is -2.5 m. If the shift is directly due to an error in the origin of the Doppler Tracking Station Coordinate system, this shift means that a value of 2.5 m should be added to the z component of the station coordinates. In a report by Marsh, (1980) the existence of a 4 m difference between Goddard Space Flight Center and the Naval Surface Weapons Center, z components of tracking station coordinates is presented. The -2.5 m shift determined in this paper is a possible explanation for part of that difference.

BIBLIOGRAPHY

Anderle, Richard J., *Accuracy of Mean Earth Ellipsoid Based on Doppler, Laser, and Altimeter Observations*, NSWC TR 80-4, (Dahlgren, Va., 1980).

Colquitt, E. S., Malyevac, C. W., and Anderle, R. J., "Doppler Computed SEASAT Orbits," *Journal of the Astronautical Sciences, Vol. XXVIII*, No. 4, pp. 391-403, Oct.-Dec. 1980.

Groeger, Walter J., *A Geoid Height and Gravity Anomaly Computer Program For Geodetic Parameter Solutions*, NWL Technical Report TR-2223 (Dahlgren, Va., April 1968).

Heiskanen, Weikko A. and Helmut Moritz, *Physical Geodesy*, W. H. Freeman and Company, San Francisco, 1967.

Lerch, F. J., Wagner, C. A., Klosko, S. M., Belott, R. P., Laubscher, R. E., and Taylor, W. A., *Gravity Model Improvement Using GEOS-3 (GEM10A and 10B)*, Paper presented at the Spring Annual Meeting of the American Geophysical Union, Miami, Florida, April 1978.

Marsh, James G., and Williamson, Ronald G., "Precision Orbit Analyses in Support of the SEASAT Altimeter Experiment," *The Journal of the Astronautical Sciences, Vol. XXVIII*, No. 4, pp. 345-369, Oct.-Dec. 1980.

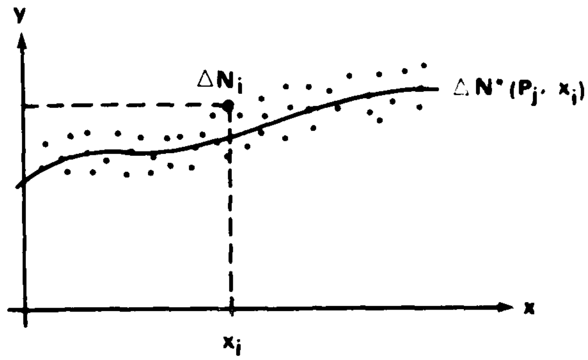
Seppelin, Thomas O., "The Department of Defense World Geodetic System 1972," Presented at the International Symposium on Problems Related to the Redefinition of North American Geodetic Networks, Frederiction, New Brunswick, Canada, May 1974.

West, G. B., Hicks, T. I., and Manrique, R. B., *NSWC/DL Filtering of GEOS-3 Radar Altimetry Data*, NSWC/DL TR-3686, (Dahlgren, Va., 1977).

APPENDIX A

LEAST-SQUARES FITTING PROCEDURES

LEAST SQUARE FITTING PROCEDURE



The given data points, ΔN_i, are defined as

$$\Delta N_i(x_i) = N_{Ai}(x_i) - N_{Si}(x_i)$$

the difference between the altimeter geoid heights, and geoid heights derived from potential coefficients at location $x_i = x_i\ (\phi_i, \lambda_i)$ for $i = 1,2,\ldots n$ where n is the total number of observations. These points, ΔN_i, are fitted to the geoid height difference function, ΔN^*, given below.

$$\Delta N^*(P_j,x_i) = \Delta a + \cos\phi_i \cos\lambda_i \Delta x + \cos\phi_i \sin\lambda_i \Delta y + \sin\phi_i \Delta z \qquad \text{(A-1)}$$

where $P_j (j=1,2,3,4)$ are the ellipsoidal parameters; Δa, Δx, Δy, and Δz, respectively. These are increments to the radius and origin components of the reference ellipsoid for the geoid derived from potential coefficients.

The purpose here is to obtain the best estimates for the unknown ellipsoid parameters stated above.

$$\Delta N_i = \Delta N^*(P_j,x_i) + e_i \qquad \text{(A-2)}$$

The "Principle of Least Squares" criterion for selecting a particular solution to Equation (A-2) is to determine the best (P_j), which minimizes the sum of errors squared after the solution

$$S = \sum_{i=1}^{n} W_i e_i^2 = \sum_{i=1}^{n} W_i[\ \Delta N_i - \Delta N^*(P_j, x_i)]^2$$

W_i is the weight matrix as defined by the number of geoid height values used in computing the mean geoid height for the i^{th} 5° x 5° area.

At this point it is convenient to restate the problem in matrix notation. Given the equation

$$\Delta N = A\Delta P + E$$

with weight matrix W and $A = \frac{\partial \Delta N^*}{\partial P}$. ΔN is the observed data, ΔN^* is the computed data, and ΔP represents the ellipsoid parameters, P_j. Find the particular solution that minimizes the sum of errors squared after the solution,

$$S = E^T W E = (\Delta N - A\Delta P)^T W (\Delta N - A\Delta P)$$

the solution[1] is given by

$$\Delta P = (A^T W A)^{-1} A^T W \Delta N$$

where

$$B = A^T W A$$

$$C = A^T W \Delta N$$

$$\Delta P = B^{-1} C$$

---

[1] John L. Junkins, "An Introduction to Optimal Estimation of Dynamical Systems" (Netherlands, Sijthoff and Noordhoff International Publishers, B. V. Alphen aan den Rijn, 1978).

In expanded form, the matrices are given as follows:

$$A_{nxj} = \begin{pmatrix} 1 & \cos\phi_1 \cos\lambda_1 & \cos\phi_1 \sin\lambda_1 & \sin\phi_1 \\ 1 & \cos\phi_2 \cos\lambda_2 & \cos\phi_2 \sin\lambda_2 & \sin\phi_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cos\phi_n \cos\lambda_n & \cos\phi_n \sin\lambda_n & \sin\phi_n \end{pmatrix}$$

$$W_{nxn} = \begin{pmatrix} W_1 & - & - & \cdot & \cdot & \cdot & - \\ - & W_2 & - & \cdot & \cdot & \cdot & - \\ - & - & W_3 & \cdot & \cdot & \cdot & - \\ \cdot & \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \cdot \\ - & - & - & \cdot & \cdot & \cdot & W_n \end{pmatrix}$$

$$\Delta N_{nx1} = \begin{pmatrix} N_{A1} - N_{S1} \\ N_{A2} - N_{S2} \\ \vdots \\ N_{An} - N_{Sn} \end{pmatrix}$$

$$C_{j\times 1} = \begin{pmatrix} \sum_{i=1}^{n} \Delta N_i \; W_i \\ \\ \sum_{i=1}^{n} \Delta N_i \; \cos\phi_i \; \cos\lambda_i \; W_i \\ \\ \sum_{i=1}^{n} \Delta N_i \; \cos\phi_i \sin\lambda_i W_i \\ \\ \sum_{i=1}^{n} \Delta N_i \; \sin\phi_i \; W_i \end{pmatrix}$$

$$\Delta P_{j\times 1} = \begin{pmatrix} \Delta a \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

The B matrix, known as the "Normal Equations," is symmetric hence only the upper triangular portion is given below.

$$B_{11} = \sum_{i=1}^{n} W_i \qquad B_{22} = \sum_{i=1}^{n} \cos^2\phi_i \cos^2\lambda_i W_i \qquad B_{34} = \sum_{i=1}^{n} \sin\phi_i \sin\lambda_i W_i$$

$$B_{12} = \sum_{i=1}^{n} \cos\phi_i \cos\lambda_i W_i \quad B_{23} = \sum_{i=1}^{n} \cos^2\phi_i \cos\lambda_i \sin\lambda_i W_i \quad B_{44} = \sum_{i=1}^{n} \sin^2\phi_i W_i$$

$$B_{13} = \sum_{i=1}^{n} \cos\phi_i \sin\lambda_i W_i \quad B_{24} = \sum_{i=1}^{n} \sin\phi_i \cos\phi_i \cos\lambda_i W_i$$

$$B_{14} = \sum_{i=1}^{n} \sin\phi_i W_i \qquad B_{33} = \sum_{i=1}^{n} \cos^2\phi_i \sin^2\lambda_i W_i$$

n is the number of 5° x 5° areas and j is the number of parameters.

DISTRIBUTION

Defense Technical Information Center Cameron Station Alexandria, VA 22314	(12)
Library of Congress ATTN: Gift and Exchange Division Washington, DC 20540	(4)
Director Defense Mapping Agency U.S. Naval Observatory Bldg. 56 ATTN: O. W. Williams C. Martin P. M. Schwimmer W. Senus R. Berkowitz Washington, DC 20360	
Defense Mapping Agency Aerospace Center ATTN: R. Ballew L. B. Decker M. Schultz K. Nelson St. Louis, MO 63118	(5)
Defense Mapping Agency Hydrographic/Topographic Center ATTN: H. Heuerman Washington, DC 20315	
Department of Geodetic Science Ohio State University ATTN: R. Rapp Columbus, OH 43210	
NOAA - National Ocean Survey/National Geodetic Survey 6001 Executive Blvd. ATTN: B. H. Chovitz B. Douglas C. Goad J. Diamante Rockville, MD 20852	

Local:

K05	
K10	(20)
K11	(5)
K12	(5)
K13	(5)
X210	(2)
X211	(2)

END

DATE
FILMED

8 82

DTIC